Dr Pankaj kumar Dorsr 16/11/2023 (1)

National Institute of Technology Hamirpur
End Semester Examination, November 2023
Class: B.Tech. (Second Year), Semester: III
Course Name: Engineering Mathematics-III
Course Code: MA-203

Time: 3 Hours **Max. Marks: 50**

Note: Each question is compulsory.

1. Sum the series

$$\sin^2\theta - \frac{1}{2}\sin 2\theta \sin^2\theta + \frac{1}{3}\sin 3\theta \sin^3\theta - \frac{1}{4}\sin 4\theta \sin^4\theta + \cdots \infty.$$

[5]

2. Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though Cauchy-Riemann equations are satisfied thereof. [5]

3. (i) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region $1 < |z| < 2$. [2]
   (ii)Find the value of $\int_C \frac{3z^2+z}{z^2-1}dz$. If C is circle $|z-1| = 1$. [3]

4. Use Bessel's formula to find the value of y if x=3.75, given [5]

x	2.5	3.0	3.5	4.0	4.5	5.0
y	24.145	22.043	20.225	18.644	17.262	16.047

5. The following data gives the melting point of an alloy of lead and zinc, where t is temperature in degree and p is the percentage of lead in the alloy.

p(%)	60	70	80	90
t	226	250	276	304

Find the melting point of the alloy containing 84% of lead, using Newton's interpolation formula. [5]

6. A canal having length 25km and width 50m is used to discharge waste water in a city. Initially the distribution of the depth of the canal (in a cross-section) from one bank is shown in the following table:

Width x (in m)	0	5	10	15	20	25	30	35	40	45	50
Depth d (in m)	0	3	7	10	15	17	15	10	7	3	0

After 25 years, it is observed that sediment deposited inside the canal and the depth distribution is also changed as shown in the following table:

Width x (in m)	0	5	10	15	20	25	30	35	40	45	50
Depth d (in m)	0	2	5	7	9	9	9	7	5	2	0

The corporation of the city is now planning to clean the canal. Estimate the total cost of removal of sediment from the canal when the cost of removal for unit volume (in m^3) is 10 Rs. [7]

7. Use the following table to compute $\int_4^{5.2} log(x)dx$ by Simpson's $\frac{3}{8}$th rule. [3]

x	4	4.2	4.4	4.6	4.8	5.0	5.2
log (x)	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487

8. Solve the following initial value problem in the range $0 \leq x \leq 0.2$ using modified Euler's method. Take $h = 0.1$.

$$\frac{dy}{dx} = y - \frac{2x}{y}, \quad y(0) = 1.$$

[5]

9. Use Milne's method to find y(1.4) from,

$$\frac{dy}{dx} = x^2(1 + y), \quad y(1) = 1.$$

Find the initial values y(1.1), y(1.2), y(1.3) using Euler's method. [5]

10. Apply the iterative method to find the real roots of $x^3 + x^2 - 1 = 0$, assuming the initial approximation is as $x_0 = 0.8$. [5]

********** **The End** **********