Prof Sunil, DoMSC

18/12/2023 (M)

National Institute of Technology Hamirpur
End Semester Examination, December 2023
Class: B.Tech. (First Year), Semester: I
Course Name: Linear Algebra and Calculus
Course Code: MA-111

Time: 3 Hours | Max. Marks: 50

Note: Each question is compulsory.

1. If the system of equations $x + ay + az = 0$, $bx + y + bz = 0$, $cx + cy + z = 0$, where a, b, c are non-zero and non-unity, has non-trivial solution then show that,

$$\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = -1$$

Also find the solution when $a = b = c = 2$. [3+2]

2. Prove that eigen values of Hermitian matrix are real. [5]

3. State the Taylor's theorem for function of two variables. Expand $f(x, y) = \tan^{-1} xy$ in power of $(x-1)$ and $(y+1)$ up to third degree term. Hence compute $f(1.1, -0.9)$ approximately. [1+3+1]

4. Trace the curve $x^2y^2 = a^2(y^2 - x^2)$. [5]

5. Evaluate the following integral by changing the order of integration to spherical coordinates [5]

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\, dy\, dz}{\sqrt{1 - x^2 - y^2 - z^2}}$$

6. Evaluate the following integral by changing the order of integration [5]

$$\int_0^2 \int_1^{e^x} dx\, dy$$

7. write and prove Frenet's formulae. [1+4]

8. (i) Find the directional derivative of $f(x, y, z) = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the vector $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. [2]
(ii) Prove that $\triangledown r^n = nr^{n-2}\mathbf{R}$, where $\mathbf{R} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$. [3]

9. State Green's theorem. Verify Green's theorem for $\int_C (xy + y^2)\, dx + x^2\, dy$, where C is bounded by $y = x$ and $y = x^2$. [1+4]

10. Evaluate $\int_S \overrightarrow{\mathbf{F}} \cdot \hat{\mathbf{n}}\, ds$ where $\overrightarrow{\mathbf{F}} = x\hat{\mathbf{i}} - y\hat{\mathbf{j}} + (z^2 - 1)\hat{\mathbf{k}}$ and S is closed surface bounded by plane $z = 0$ and $z = 1$ and cyclinder $x^2 + y^2 = 4$. Also verify Gauss-Divergence theorem. [3+2]

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