# National Institute of Technology, Hamirpur Department of Computer Science and Engineering 

## B.Tech. DD (Section B) 5th Sem (End-term Examination)

CS - 311 Analysis and Design of Algorithms

Time: 3 hrs
Max. Marks 50

## Instructor: Nitin Gupta

For all problems, expand every function that your are calling. Further, if you include any new algorithm in your solution, please also include a brief English description of what the algorithm does.

1. (8 Marks) In a knapsack problem we are given positive integers $p_{1} \ldots p_{n}, w_{1} \ldots w_{n}$ and m . Find $x_{1} \ldots x_{n}$ such that it

$$
\begin{gathered}
\operatorname{maximize} \sum_{1 \leq i \leq n} p_{i} x_{i} \\
\text { subject to } \sum_{1 \leq i \leq n} w_{i} x_{i} \leq m \\
\text { and } 0 \leq x_{i} \leq 1 .
\end{gathered}
$$

Write algorithm for above mentioned knapsack problem such that it return $x[1 \ldots n]$. Perform your algorithm over set of values as given: $n=7 ;\left(P_{1}, P_{2}, \ldots, P_{7}\right)=\{10,5,15,7,6,18,3\}$; $\left(w_{1}, w_{2}, \ldots, w_{7}\right)=\{2,3,5,7,1,4,1\}$ and $m=15$.
2. (10 Marks) Give algorithm for Quick sort and apply the same to sort the list C, O, M, $P, U, T, E, R$ in alphabetic order. What value of $q$ does PARTITION return when all elements in the array $\mathrm{A}[\mathrm{p} \ldots \ldots \mathrm{r}]$ have the same value?
3. (12 Marks) Suppose we are given $n$ distinct positive numbers and we desire to find all combinations of these numbers whose sum are $m$. Write algorithm to find the solution. Let $w=\{12,5,10,15,13,18\}$ and $\mathrm{m}=30$. Find all the possible subsets of w that sum to m . Create a pruned state space tree that is generated. How can you prove that the problem is NP-complete?
4. (10 Marks) A map is shown in Fig. 1 with five regions. Draw a corresponding planer graph for this map and find out the chromatic number of that graph. Now, suppose that you assigned an integer between $\{1 \ldots n\}$ to color the node, then weight of the edge ( $u, v$ ) is sum of the integers assigned to $u$ and $v$ res. Apply Prim's algorithm on this graph to find the Minimum Spanning Tree of the graph.

## What is the chromatic number for the map?



Figure 1: Map
5. (10 Marks) Consider the directed graph in figure 2. Write Breadth First Search algorithm and apply that on the given figure to show the working of your algorithm. Further, draw BFS tree for the same. Can you apply Topological Sort on this graph? Why?


Figure 2: Directed Graph
Hz

