Dr Talari Ganesh 28/11/2023

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NATIONAL INSTITUTE OF TECHNOLOGY, HAMIRPUR (HP)– 177 005

End Semester Examination, November 2023

MA-212: Stochastic Processes

Time: 3 Hrs **Max Marks: 50**

Attempt all questions

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1. (a) Suppose that the random variables X and Y have a joint density function given by $f(x,y)=c(2x+y),\ \ 0<x<6,\ 0<y<5$ then find (a) the constant c, (b) the marginal distribution functions for X and Y, (c) the marginal density functions for X and Y, (d) P (3<X<4, Y>2). **(5)**

   (b) Let (X, Y) be a two-dimensional non-negative continuous random variable having the joint density given below. Test whether X and Y are independent and find the conditional density of X given Y = y, $f(x,y)=4xy\, e^{-(x^2+y^2)}, x\geq 0, y\geq 0$ **(5)**

2. (a) Let X_1, X_2 be independent random variables each having geometric distribution, then show that the conditional distribution of X_1 given $X_1 + X_2$ is uniform distribution. **(5)**

   (b) Show that the recurrence relation for the moments of Normal distribution is $\mu_{2n}=\sigma^2(2n-1)\mu_{(2n-2)}$ **(5)**

3. (a) The daily consumption of milk in a city in excess of 20,000 gallons is approximately distributed as Gamma variate with parameter α=2 and β=0.0001. The city has a daily stock of 30,000 gallons, and then what is the probability that the stock is insufficient on a particular day? **(5)**

   (b) If X is Exponential distributed with parameters λ, then find the value of k such that $P(X>k)|P(X\leq k)=a$ **(5)**

4. (a) Consider $\{X_n, n\geq 0\}$ be a Markov Chain with three states $\{0,1,2\}$ has the following transition matrix A and the initial distribution is (0.5, 0.3, 0.2), then find (i) $P(X_3=2, X_2=1, X_1=0, X_0=2)$, (ii) $P(X_3=2, X_1=0, X_0=2)$, (iii) $P(X_3=2, X_1=0)$, (iv) $P(X_2=1)$ **(5)**

$$A=\begin{bmatrix} 0.2 & 0.3 & \\ & 0.6 & 0.3 \\ 0.4 & & 0.3 \end{bmatrix}$$

   (b) If be a Poisson process with rate λ>0, then prove that the random variable Y describing the number of events in any time interval of length t>0 has a Poisson distribution with parameter λt. **(5)**

5. (a) Derive the steady state equations for the model (M/M/C): (N/FCFS), and also derive p_0, p_n, L_q, & average number in the service facility ($L_s - L_q$). **(5)**

   (b) A telephone exchange has two long distance operators. The telephone company finds that, during the peak load, long distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately Exponential distributed with mean length 5 minutes. If the subscriber waits for his turn, what is the expected waiting time? **(5)**